

Quantum Illumination

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Abstract: The use of entangled light to illuminate objects is shown to provide significant enhancements over unentangled light for detecting and imaging those objects in the presence of high levels of noise and loss. Each signal sent out is entangled with an ancilla, which is retained. Detection takes place via an entangling measurement on the returning signal together with the ancilla. Quantum illumination with e bits of entanglement increases the effective signal-to-noise ratio of detection and imaging by a factor of 2^e , an exponential improvement over unentangled illumination.

We are trying to detect the presence of an object by shining light in its direction and seeing if any is reflected back. Only a small percentage of the light we shine is reflected. The object is immersed in noise and thermal radiation, so we have to distinguish whatever light is reflected from the noisy background. The usual solution to this problem, as in, e.g., radar or lidar, is to send a signal into the region where the object may be, and to monitor the radiation returning from that region to see if any trace of the signal can be detected above the background noise. Typically, the great majority of the signal is lost and only a tiny fraction returns, if any.

In the case of quantum bits, it is known that the sensitivity of detection processes can be enhanced by entangling the signal with an ancilla, and by making an entangling measurement on the returning radiation together with that ancilla [1]. Here, we ask whether entanglement gives an enhancement of sensitivity for quantum optical processes, and investigate the amount of such enhancement, if any. The intuition is that if we send out a

signal photon that is entangled with an ancillary photon, then when that photon comes back, if ever, it will prove easier to recognize as the same photon that was sent out. This intuition will turn out to be correct, *even though noise and loss completely destroy the entanglement between signal and ancilla*. Moreover, the entanglement-induced enhancement of sensitivity is significant: it grows exponentially with the number of bits of entanglement between signal and ancilla. The exponential enhancement in effective signal-to-noise ratio persists when we process returning light to construct an image of the object.

In this paper, we present the simplest possible treatment of quantum illumination, where signal and ancilla consist of individual photons created, e.g., via spontaneous parametric downconversion. A more complete treatment of quantum illumination in terms of generalized Gaussian states will be presented elsewhere. Suppose that we send a single photon at the object to be detected. If the object is not there, the signal that we receive back consists of thermal and background noise. Even if the object is there, the great majority of time the photon is lost, and we receive only noise. Once in a great while, the photon is reflected to us in a perturbed form. The dynamics corresponding to this situation can be modelled as beam splitter with reflectivity η , that mixes in the vacuum with the signal state, followed by thermalization which mixes in noise with average photon number b per mode. No object corresponds to a beam splitter with $\eta = 0$. The presence of an object corresponds to a beam splitter with a very small non-zero reflectivity.

We repeatedly send single photons at the object, and try to detect reflected photons. The number of noise photons per mode will be taken to be small. This scenario corresponds, e.g., to single photons in the optical regime directed at a distant target which is bathed in thermal radiation at temperature significantly below optical energies. In this case, $b = (1 - e^{-\hbar\omega/kT})e^{-\hbar\omega/kT}$ is the average number of thermal photons per mode. Our detector can distinguish between d modes per detection event: $d = WT$, where W is the bandwidth of the detector and T is the length of the detection window. We take the time window for detection sufficiently small that at most one noise photon is detected at a time, i.e., $db \ll 1$. Additional, non-thermal noise can be tolerated as well, as long as fewer than one photon arrives per detection event.

First, consider the case of unentangled light. We send a single photon in the state ρ toward the region where the object might be. The two different dynamics corresponding to object there and object not there are as follows [2]:

Case (0), object not there: $\rho \rightarrow \rho_b \otimes \dots \rho_b$, where ρ_b is the thermal state with b photons

per mode. Because the average number of photons bd received per detection event is much less than 1, we can approximate the thermal state as

$$\rho_0 = (1 - db)|vac\rangle\langle vac| + b \sum_{k=1}^d |k\rangle\langle k|. \quad (1.0)$$

Here, $|vac\rangle$ is the vacuum state of the modes, and $|k\rangle$ is the state where there is a single photon in mode k and no photons in the other modes. Since $\|\rho_0 - \rho_b \otimes \dots \rho_b\|_1 = (db)^2 + O((db)^3)$, we can safely replace the exact thermal state by ρ_0 as long as we are evaluating expression to lowest order in db .

Case (1), object there:

$$\rho \rightarrow (1 - \eta)\rho_0 + \eta\rho_{th},$$

where ρ_{th} is the thermalized version of ρ . It is straightforward to verify that $\|\rho - \rho_{th}\| = db + O((db)^2)$. Accordingly, if we are not interested in terms of $O(\eta db)$, then we can safely approximate $(1 - \eta)\rho_0 + \eta\rho_{th}$ as

$$\rho \rightarrow \rho_1 = (1 - \eta)\rho_0 + \eta\rho. \quad (1.1)$$

Now we repeatedly send single photons in the state $|\psi\rangle$ to detect the presence or absence of the object. We look at the signal coming back to try to determine whether the object is there or not. The single shot minimum probability of error is obtained by projecting onto the positive part of $\rho_1 - \rho_0$ (see reference [2] for a treatment of discrimination between different dynamics in the case of qubits). This measurement consists simply of verifying whether a returning photon is the state $|\psi\rangle$ or not. If the measurement yields a positive result, then we guess that the object is there. If the result is negative, then we guess that the object is not there. The conditional probabilities of the outcomes ‘yes’ and ‘no’ given the presence or absence of the object are

$$\begin{aligned} p(\text{no}|\text{not there}) &= 1 - b & p(\text{no}|\text{there}) &= (1 - b)(1 - \eta) \\ p(\text{yes}|\text{not there}) &= b & p(\text{yes}|\text{there}) &= b(1 - \eta) + \eta. \end{aligned} \quad (2)$$

The number of trials required to reveal the presence or absence of the object depends on the ratio η/b . If $\eta/b > 1$, then a received photon is more likely to be a signal photon than a noise photon: the signal to noise ratio is greater than 1. Call this regime, the good regime. Similarly, if $\eta/b < 1$, then a received photon is more likely to be a noise photon

than a signal photon: the signal to noise ratio is less than 1. Call this regime, the bad regime. In the good regime, the probability that no photons have been received after n trials, given that the object is not there, goes as $(1 - b)^n$. the probability that no photons have been received after n trials, given that the object is there, goes as $(1 - \eta)^n$. That is, the number of trials required to detect the object, if there, goes as $1/\eta$: one simply sends photons until one receives one back. If the object is there, then one receives a photon back considerably before one would expect a photon given thermal photons alone.

In the bad regime, most of the photons received are noise photons. Here, one must count photons until one can separate the thermal distribution with b photons on average, from the distribution when the object is there, which has $b + \eta$ photons on average. Using the usual formulae for sampling the outcomes of Bernoulli trials, one finds that takes on the order of $8b/\eta^2$ photons on average to distinguish between the presence or absence of the object in the bad regime.

The optimal asymptotic minimum probability of error as the number of trials n gets large, is given by the quantum Chernoff bound [3]: $p_n(\text{error}) \approx (1/2)Q^n$, where

$$Q = \min_s \text{tr} \rho_0^{1-s} \rho_1^s. \quad (3)$$

To lowest order in η, b , we have

$$\rho_0^{1-s} = (1 - db)^{1-s} |vac\rangle\langle vac| + b^{1-s} I, \quad (4.0)$$

where $I = \sum_k |k\rangle\langle k|$ is the identity operator on the single photon subspace. Similarly,

$$\rho_1^s = (1 - bd - \eta)^s |vac\rangle\langle vac| + b^s (I - |\psi\rangle\langle\psi|) + (b + \eta)^s |\psi\rangle\langle\psi|. \quad (4.1).$$

The trace of $\rho_0^{1-s} \rho_1^s$ can be readily evaluated, and the quantum Chernoff bound takes the form

$$Q = \min_s \left(1 - \eta s + b \left(-1 + \left(1 + \frac{\eta}{b} \right)^s \right) \right) + O(b^2, \eta b). \quad (5)$$

Just as in the iterated single shot case, equation (5) implies the existence of two regimes for evaluating the quantum Chernoff bound, depending on whether $\eta > b$ (good) or $\eta < b$ (bad). The exact value of s that minimizes Q in the good regime depends on the ratio b/η . In the limit that $b/\eta \ll 1$, the minimum occurs for $s \rightarrow 1$, and

$$Q = 1 - \eta + O((db)^2) \quad (6)$$

Comparing the quantum Chernoff bound with the minimum error single shot bound above, we see that they yield the same asymptotic error probability. Accordingly, as before, the optimal measurement needed to attain the quantum Chernoff bound is simply to count the number of signal photons in the state $|\psi\rangle$ that return. In the good regime, if the object is there, such a photon will come back after $1/\eta$ trials on average, sooner than the $1/b$ trials expected if the object is not there.

When $\eta < b$, a photon that comes back is more likely to be a noise photon than a signal photon. As before, the $\eta < b$ regime is the ‘bad’ regime where the signal to noise ratio less than one. Evaluating equation (5) in this regime, we find

$$Q = 1 - \frac{\eta^2}{8b} + O(\max b^2, \eta b). \quad (7)$$

(Note that $\eta^2/b \gg \eta b, b^2$, because $b \ll 1$.) Once again, the quantum Chernoff bound coincides asymptotically with the single shot minimum error rate: the optimal measurement is simply to count returning photons in the state $|\psi\rangle$ to see if the number of photons returned is significantly different from the number expected if we were receiving only thermal radiation. Here, because of the low signal to noise ratio, the signal gives only a small shift in the average number of received photons, and $\approx 8b/\eta^2$ photons must be sent to detect the object with high probability.

Note that the error probabilities for detecting the presence or absence of the object do not depend on the number of signal and detector modes d . The number of modes doesn’t matter because all modes other than the one in which the photon is sent are in a thermal state. These other modes give us no information about the presence or absence of the object. To detect the object, we need only monitor the mode in which we sent the photon to see if more than the expected number of photons come back.

Sending entangled photons

Now let us look at the effect of entanglement on our ability to detect the object. Construct the entangled state $|\psi\rangle_{SA} = (1/\sqrt{d}) \sum_k |k\rangle_S |k\rangle_A$ for signal photon and ancilla photon, and send the signal photon to towards where the object is likely to be. This state can be constructed, for example, by taking the output of a spontaneous parametric downconverter in the low-photon number regime, matching its time-bandwidth product to the time-bandwidth product of the detector, and selecting out the one signal photon/ one idler photon sector (as will be seen below, photodetection at the receiver can automatically

postselect this state). The signal photon is sent off and the idler photon is retained as the ancilla.

The two different dynamics corresponding to object or no object now take a slightly different form, as the state of the ancilla must be included. If the signal photon is lost, the ancilla photon goes to the completely mixed state:

Case (0), object not there:

$$|\psi\rangle_{SA}\langle\psi| \rightarrow \rho_{SA0} = \rho_0 \otimes \frac{I_A}{d} = ((1 - db)|vac\rangle_S\langle vac| + bI_S) \otimes \frac{I_A}{d} + O(b^2). \quad (8.0)$$

Case (1), object there:

$$|\psi\rangle_{SA}\langle\psi| \rightarrow \rho_{SA1} = (1 - \eta)\rho_{SA0} + \eta|\psi\rangle_{SA}\langle\psi| + O(b^2, \eta b). \quad (8.1)$$

As before, the single-shot minimum error probability is obtained by projecting onto the positive part of $(\rho_{SA1} - \rho_{SA0})$, which in turn simply corresponds to determining whether any returning photon is in the state $|\psi\rangle_{SA}$.

A detailed treatment of entangled photodetection lies outside the scope of this paper. Although such entangling measurements are likely to prove difficult, they are certainly allowed by the laws of physics. Take the case where $|\psi\rangle_{SA}$ is the single-photon pair output of a spontaneous parametric downconverter, for example. In this state, the signal and idler modes are anticorrelated in momentum, so that $\omega_k^S + \omega_k^I = \omega$, where ω is the pump frequency. In addition, because of the form of the entanglement, the signal and idler modes are correlated in time of arrival, so that both photons will arrive at a receiver at the same time.

To check if the two photons are in the state $|\psi\rangle_{SA}$, one needs to verify both frequency anticorrelation and time of arrival correlation between the photons. More explicitly, the entangling measurement on signal and ancilla must verify that signal and ancilla frequencies sum to ω , without distinguishing between the different modes k ; then photodetection must be carried out to check that both signal and ancilla arrive at the detector at the same time. For example, the detector could consist of an ensemble of atoms which can only make two-photon transitions with sum frequency ω . Because of the positive correlation in time of arrival, the absorption rate of the two entangled photons is linear in the photon flux density rather than quadratic [4] – the entangled state is much more likely to induce such

a transition than an unentangled state. The ensemble can then be queried using, e.g., a cycling transition, to determine if any two-photon transition has taken place. Note that such two-photon detection retroactively post-selects the single photon pair state out of the spontaneous parametric amplifier output state in the low flux regime.

Making the optimal single-shot measurement and evaluating the conditional error probabilities for the entangled case yields

$$\begin{aligned} p_e(\text{no}|\text{not there}) &= 1 - \frac{b}{d} & p_e(\text{no}|\text{there}) &= (1 - \frac{b}{d})(1 - \eta) \\ p_e(\text{yes}|\text{not there}) &= \frac{b}{d} & p_e(\text{yes}|\text{there}) &= (1 - \eta)\frac{b}{d} + \eta. \end{aligned} \quad (9)$$

It is immediately seen that the effect of entanglement is to reduce the effective noise from b to b/d . This reduction reflects the fact that in the entangled case a noise photon together with the fully mixed ancilla is d times less likely to be confused for a signal photon entangled with the ancilla, than a noise photon is likely to be confused with a signal photon in the unentangled case. Entanglement reduces the effective signal to noise by a factor of d .

The single-shot minimum error probability for the entangled case coincides with the asymptotic minimum error probability by evaluating the quantum Chernoff bound, $Q = \min_s \text{tr} \rho_{SA0}^{1-s} \rho_{SA1}^s$. The roots of the density matrices can be evaluated and are only slightly more complicated than before:

$$\begin{aligned} \rho_{SA0}^{1-s} &= \rho_S^{1-s} \otimes \frac{I_A}{d^{1-s}} \\ &= ((1 - db)^{1-s} |vac\rangle_S \langle vac| + b^{1-s} I_S) \otimes \frac{I_A}{d^{1-s}}. \end{aligned} \quad (10.0)$$

$$\begin{aligned} \rho_{SA1}^s &= ((1 - \eta)\rho_{SA0} + \eta|\psi\rangle_{SA}\langle\psi|)^s \\ &= (1 - bd - \eta)^s |vac\rangle_S \langle vac| \otimes \frac{I_A}{d^s} \\ &\quad + (1 - \eta)^s \left(\frac{b}{d}\right)^s (I_S \otimes I_A - |\psi\rangle_{SA}\langle\psi|) + ((1 - \eta)\frac{b}{d} + \eta)^s |\psi\rangle_{SA}\langle\psi|. \end{aligned} \quad (10.1)$$

Taking the trace of $\rho_{SA0}^{1-s} \rho_{SA1}^s$, we obtain

$$\begin{aligned} Q &= \min_s (1 - \eta)^s \left(1 + \frac{b}{d} \left(-1 + \left(1 + \frac{\eta d}{(1 - \eta)b} \right)^s \right) \right) \\ &= \min_s \left(1 - \eta s + \frac{b}{d} \left(-1 + \left(1 + \frac{\eta d}{b} \right)^s \right) \right) + O(b^2, \eta b). \end{aligned} \quad (11)$$

Equation (11) confirms that the effect of entanglement is to reduce the effective noise from b to b/d .

Comparing the quantum Chernoff bound for entangled states, equation (9), the quantum Chernoff bound for unentangled states, equation (4), we see that there are once more two regimes. The good regime now occurs when $\eta d/b > 1$. In this regime, the quantum Chernoff bound is

$$Q \approx 1 - \eta \quad (11)$$

in the limit $\eta d/b \gg 1, s \rightarrow 1$. Comparing the entangled case to the unentangled case above, we see that the quantum Chernoff bound is the *same* in the good regime in both cases, but the good regime extends d times further in the entangled case than in the unentangled case, where the good regime occurred for $d/b > 1$.

The extension of the good regime via the use of entangled photons can be understood as follows. As before, the quantum Chernoff bound coincides with repeated single-shot minimum error measurements, showing that the optimal detection strategy is to measure any incoming photon together with the ancilla to see if the two photons are in the state $|\psi\rangle_{SA}$. If the photon that returns is the signal photon, then it will pass the test. If the photon that returns is a noise photon, then the ancilla is in the state I_A/d , and the noise photon together with fully mixed ancilla are d times less likely to be found in the state $|\psi\rangle_{SA}$. A noise photon in the entangled case is d times less likely to pass the test and be confused as a signal photon than a noise photon in the unentangled case. In other words, the presence of entanglement makes it d times harder for a noise photon to masquerade as a signal photon. Entanglement effectively enhances the signal to noise ratio by a factor of d .

The bad regime for the entangled case occurs for $\eta d/b < 1$. In this case the quantum Chernoff bound occurs for $s \approx 1/2$ and is

$$Q = 1 - \frac{\eta^2 d}{8b} + O(b^2, \eta b). \quad (13)$$

Comparing with equation (7) for the quantum Chernoff bound in the unentangled bad regime, we see that the entangled bound is d times better than the unentangled bound: quantum illumination reduces the number of trials needed to detect the object by a factor d . Entanglement effectively enhances the signal to noise ratio by the degree of entanglement, even in the bad regime.

The fact that entanglement yields an enhancement in the bad regime is particularly interesting, because in the bad regime the combination of noise and loss insures that *no entanglement between signal and ancilla survives*, an effect that also appears in the qubit

case [1]. Nonetheless, even though signal and ancilla are unentangled at the detector, a noise photon still finds it d times harder to masquerade as a signal photon entangled with an ancilla photon.

Imaging

Entanglement effectively enhances the signal to noise ratio of detection by a factor of d , where d is the number of entangled modes. Measured in terms of e = the number of e-bits of entanglement, the enhancement is

$$d = 2^e. \quad (14)$$

The enhancement is exponential in the number e-bits.

This enhancement persists in imaging. Suppose that one images an object by illuminating it in a point-by-point fashion. Light is focused on the object to the minimum spot size allowed by the Rayleigh bound. Light reflecting from that spot is focused on an image plane using a lens. The dimensions of the spot's image are also determined by the Rayleigh bound. Photodetection and postprocessing of the signal at the image plane then allow the amount of light reflected from that spot on the object to be measured. The light is now focused on another spot and the process is repeated until the entire object has been imaged. Such a method can yield a classical enhancement of \sqrt{n} over the Rayleigh limit, where n is the number of photons collected for each point [5-6].

A key limitation of this semiclassical imaging method is the presence of noise in the form of light originating from points other than the point that is being imaged at that moment. If the light used to image the object is entangled with an ancilla in the fashion described above, and entangled detection of signal and ancilla is performed at the image plane, then once again noise photons find it d times harder to masquerade as signal photons. The same quantum illumination techniques that allow an exponential enhancement of the signal to noise ratio for detection also provide a potential exponential enhancement of imaging. In imaging, however, the Rayleigh limit on the spatial extent of the modes at the image plane, together with fact that the entangled photo-detection is a two photon process raise serious questions about the efficiency of photodetection, despite the positive correlation in photon time of arrival [4]. Considerable further analysis of multiple quantized spatial modes must be performed to identify the scenarios in which quantum illumination provides an actual practical advantage over ordinary illumination for imaging.

Discussion

Quantum illumination is a potentially powerful technique for performing detection and imaging, in which signal is entangled with an ancilla, and entangling measurements are made at the detector. Entanglement enhances the effective signal to noise ratio because a noise photon has a d times harder time masquerading as an entangled signal photon, compared with a noise photon masquerading as an unentangled signal photon. The enhancement of sensitivity and effective signal-to-noise ratio that quantum illumination provides is exponential in the number of bits of initial entanglement, and persists even in the presence of large amounts of noise and loss, when no entanglement survives at the receiver. Many practical questions remain, notably, how can the requisite entangled measurements be performed efficiently? Does the enhancement persist at higher noise temperatures and for larger numbers of photons in the signal? What are the maximum enhancements obtainable via quantum illumination over all possible input states, including Gaussian states? These questions and many others must be answered before quantum illumination can prove itself useful in practice.

Acknowledgements: This work was supported by the W.M. Keck foundation and by DARPA. The author would like to thank Baris Erkmen, Vittorio Giovannetti, Saikat Guha, Lorenzo Maccone, Stefano Pirandola, Jeffrey Shapiro, Si-Hui Tan, Mankei Tsang, and Horace Yuen for useful discussions.

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